Bias correction of satellite radiances is an essential component of data assimilation system in Numerical Weather Prediction (NWP). Variational Bias Correction (VarBC) schemes are widely used by global NWP centers (NCEP, ECMWF), but there are still open questions regarding their use in limited-area models (LAMs). We shall present a study of VarBC adaptivity in the limited-area 3D-Var system using the state-of-the-art NWP system ALADIN, which shares its model code with the global system IF/ARPEGE, and is operationally used at Czech Hydrometeorological Institute.

1. Model ALADIN/CZ

   - Central Europe domain (Fig.1),
   - $\delta x \sim 4.7 \text{ km}$, 87 vertical levels up to 0.1 hPa,
   - BlendVar for upper-air fields (DF blending + 3D-Var),
   - 6-hour analysis cycle at 0, 6, 12 and 18 UTC,
   - Radiance observations: SEVIRI (Meteosat-10) ATOS/IASI (NOAA, MetOp).

2. Variational Bias Correction

   VarBC is an adaptive bias correction scheme implemented into the 3D-Var data assimilation system:
   
   \[ J(x,\beta) = \left( x - x_0 \right) B^{-1} \left( x - x_0 \right)^T + (\beta - \beta_0) B^{-1} (\beta - \beta_0) + (y - h(x, \beta))^T R^{-1} (y - h(x, \beta)) \]

   \[ \beta = \left( \begin{array} {c} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{array} \right) \]

   \( J(x,\beta) \) is the background bias parameter error covariance matrix simplified by diagonal elements:

   \[ \sigma_{\beta}^2 = \frac{\sigma_{\text{var}}^2}{N_{\text{obs}}} \]

   The lack of independence in LAM implies a higher sample variance of \( \beta \) (blue-line; Fig.3) invalidating (3). Therefore, we propose a new formulation of \( N_{\text{obs}} \) that is able to reduce the sample variance with respect to the satellite bias component. This concept is based on the variance-bias trade-off considering the Mean-Square Error (MSE) of \( \beta \) at n-th time step:

   \[ MSE(\beta_n) = \text{var}(\beta_n) + \text{bias}(\beta_n, \text{target})^2. \]

   The optimum stiffness parameter is estimated by minimizing the MSE function (Fig.4) with respect to \( N_{\text{obs}} \) such that:

   \[ N_{\text{obs}} \approx \text{avg}(N_{\text{obs}}, N_{\text{avg}}) \left[ 2nW + 4\text{var}^2 \text{var}(\beta_n) \right]^{-1}. \]

3. LAM Sampling Issues

   The observation bias is detected at each analysis time based on observation-minus-model (OMG):

   \[ b_n = (y - h(x)); \]

   Estimate of \( b_n \) is statistically meaningful provided a normal independent sample of OMG. In LAM, the sample of polar satellites is spatial/time dependent caused by:

   - non-uniform data coverage (Fig.1),
   - regional weather conditions: different NWP model biases for typical weather regimes, seasons or diurnal cycle.

   The higher sample variance of \( b_n \) is detected in LAM’s analysis cycle (see Fig.2).

4. VarBC adaptivity in LAM

   The key parameter for the VarBC adaptivity is the stiffness parameter \( N_{\text{obs}} \) (Eq.1). The default \( N_{\text{obs}} \) setting (5000) is not flexible for all satellite instruments/channels providing rapid or too slow adaptation of \( \beta \) (Fig.5). Cameron and Bell (2016) suggested a harmonized stiffness parameter provided \( N_{\text{obs}} \) independent observation sample when initial bias parameter \( \beta_0 \) decreases exponentially towards the best fit \( \beta_{\text{best}} \) (red-line; Fig.5):

   \[ \beta_0 = \left( \frac{N_{\text{obs}}}{N_{\text{obs}} + \text{target}} \right)^{n} \beta_{\text{best}} + 1 - \left( \frac{N_{\text{obs}}}{N_{\text{obs}} + \text{target}} \right) \beta_{\text{init}}. \]

   The optimum stiffness parameter is estimated by minimizing the MSE function (Fig.4) with respect to \( N_{\text{obs}} \) such that:

5. Evaluation of VarBC

   Different VarBC initialization methods are compared during a spin-up period in 09/2015 (Fig.5) during which the radiance observations do not influence the model’s initial state (passive mode). A quality of VarBC is evaluated using different \( \beta \) (after initialization) during a validation period in 10/2015 (Fig.6).

6. The bias correction quality measured by MSE of the corrected observation bias (95% confidence level) w.r.t VarBC initialization methods. Overall scores are calculated for AMSU-A (left) and MHS (middle) channels. Top-right: The MSE scores w.r.t the satellite scan-angle. Bottom-right: The VarBC response to an artificial satellite bias 0.3 K using default \( N_{\text{obs}} \) (5000) and new \( N_{\text{obs}} \). The \( K_{\text{AMSU}} \) and \( K_{\text{MHS}} \) correspond to spin-up periods \( n = 10 \) and 30.

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References:
