# CHAPTER 11

# SATELLITE ORBITS

### *11.1 Orbital Mechanics*

Newton's laws of motion provide the basis for the orbital mechanics. Newton's three laws are briefly (a) the law of inertia which states that a body at rest remains at rest and a body in motion remains in motion unless acted upon by a force, (b) force equals the rate of change of momentum for a body, and (c) the law of equal but opposite forces. For a two body system comprising of the earth and a much smaller object such as a satellite, the motion of the body in the central gravitational field can be written

$$d\vec{v} / dt = -GM \vec{r} / r^3$$

where $\vec{v}$ is the velocity of the body in its orbit, G is the universal gravitational constant (6.67 x 10**11 N m**2/kg**2), M is the mass of the earth, and r is position vector from the centre of mass of the system (assumed to be at the centre of the earth). The radial part of this equation has the more explicit form

$$m\, d^2r/dt^2 - mr\omega^2 = -GMm/r^2$$

where $\omega$ is the angular velocity and m is the satellite mass. The second term on the left side of the equation is often referred to as the centrifugal force

The total energy of the body in its orbit is a constant and is given by the sum of the kinetic and potential energies

$$E = 1/2\, m\, [(dr/dt)^2 + r^2\omega^2] - mMG/r\ .$$

The angular momentum is also conserved so that

$$L = m\omega r^2\ ,$$

which allows us to rewrite the energy

$$E = 1/2\, m\, (dr/dt)^2 + L^2/(2mr^2) - mMG/r\ .$$

The magnitude of the total energy reveals the type of orbit;

$E > 0$ implies an unbounded hyperbola,

$E = 0$ unbounded parabola,

$E < 0$ bounded ellipse,

$E = -G^2M^2m^3/(2L^2)$ bounded circle.

The last two terms of the total energy can be considered as an effective potential energy, which has a minimum for the radius of the circular orbit.

The circular orbit requires that the radius remain constant (so dr/dt equals zero) through the balance of the gravitational and centrifugal forces

$$GMm/r^2 = m\omega^2 r$$

or

$$r = L^2/(m^2MG) .$$

This leads to the expression for the total energy of the satellite in a bounded circle.

It is also worth noting that the conservation of angular momentum is directly related to Kepler's law of equal areas swept out in equal time, since

$$dA/dt = r^2\omega/2 = L/(2m) = \text{constant} .$$

The derivative with respect to time can be rewritten as a derivative with respect to angle through the conversion

$$d/dt = (L/mr^2)d/d\theta$$

so that

$$m\, d^2r/dt^2 = (L/r^2)d/d\theta[(L/mr^2)dr/d\theta]$$

and using u = 1/r this becomes

$$[d^2u/d\theta^2 + u] = GMm^2/L^2 .$$

The solution has the form of the equation for a conic section

$$1/r = [GMm^2/L^2]\,[1 + \varepsilon \cos\theta]$$

where the eccentricity $\varepsilon$ is given by

$$\varepsilon^2 = [1 + 2EL^2/(G^2M^2m^3)]$$

The point of closest approach, the perigee, occurs for $\theta = \pi$; the apogee occurs for $\theta = 0$.

## *11.2 The Geostationary Orbit*

Let us continue our discussion of the circular orbit. Using the definition of angular velocity $\omega = 2\pi/\tau$ where $\tau$ is the period of the orbit, then

$$GMm/r^2 = m\omega^2 r$$

becomes

$$GM/r^3 = 4\pi^2/\tau^2 .$$

For the geostationary orbit, the period of the satellite matches the rotational period of the earth so that the satellite appears to stay in the same spot in the sky. This implies that $\tau$ = 24 hours = 8.64x10**4 seconds, and the associated radius of the orbit r = 4.24x10**7 metres or a height of about 36,000 km. The geostationary orbit is possible at only one orbit radius.

For a polar circular orbit with $\tau$ = 100 minutes = 6x10**4 seconds, we get r = 7.17x10**6 metres or a height of about 800 km. Polar orbits are not confined to a unique radius, however the type of global coverage usually suggests a range of orbit radii.

It is useful to note that

$$g = 9.8 \text{ m/s}^2 = GM/R^2$$

where R is the radius of the earth (about 6400 km).

## *11.3 Orbital Elements*

A detailed knowledge of orbits allows the determination of what areas will be viewed by the satellite, how often, and when. Proper adjustment of the orbit can enhance repeated coverage of one area at the same time each day or repeated coverage of the same area every few minutes throughout the day. The coverage depends on six orbital elements.

To a first approximation a satellite close to the earth has an elliptical orbit. The orientation of the satellite orbit plane is described by (a) the inclination of the satellite orbit plane with respect to the earth equatorial plane denoted by I, and (b) the right ascension of the ascending node, $\Omega$, measured eastwards relative to Aries (representing a fixed point in the heavens). The shape and size of the satellite orbit is given by (c) the semi-major axis of the ellipse denoted by a, and (d) the eccentricity of the ellipse, denoted by $\varepsilon$. The orientation of the orbit in the orbit plane is given by (e) the argument of the perigee or the angle between the ascending node and the perigee denoted by w. And finally (f) $\theta$ denotes the angular position of the satellite in its orbit. These are the six orbital elements that are necessary to calculate the trajectory of the satellite in its orbit; they are shown in Figures 11.1 and 11.2.

The geostationary orbit provides good temporal (half hourly) and spatial coverage over the equatorial regions (but its viewing angle to the polar regions is poor). On the other hand the polar orbit provides good coverage of the polar regions every 100 minutes (but its viewing over the equatorial regions is incomplete and less frequent). Thus studies over the tropics and the ITCZ (Inter Tropical Convergence Zone) rely mostly on geostationary satellite data, while the Arctic and Antarctic polar studies depend primarily on polar orbiting satellite data.

The elliptical orbit is an approximation; there are several small perturbing forces which are mentioned briefly here. (a) The earth is not spherical; it exhibits equatorial bulge which perturbs the orbital elements. This is an important correction; (b) Atmospheric drag significantly slows satellites below a height of 150 km, but is very small for orbit heights greater than 1500 km; (c) Solar wind and radiation slightly influence the orbit of satellites with low density; (d) The gravitational influence of the other bodies in the solar system, particularly the sun and the moon, are small but nonzero; (e) Relativistic effects are very small.

The primary influence of the non spherical earth gravitational field is to rotate the orbital plane, to rotate the major axis within the orbital plane, and to change the period of the satellite.

## *11.4 Gravitational Attraction of Non-spherical Earth*

The earth's gravitational field is not that of a point mass, rather it is the integrated sum over the bulging earth. The potential energy for a satellite of mass m a distance r from the centre of mass of the earth is written

$$PE = - Gm \int_{\text{earth}} dM/s$$

where the integration is over the mass increment dM of the earth which is a distance s from the satellite. This integration yields a function in the form

$$PE = -GMm/r \left[1 - \sum_{n=2,...} J_n (R/r)^n P_n(\cos\theta)\right]$$

where the $J_n$ are coefficients of the $n^{th}$ zonal harmonics of the earths gravitational potential energy and the $P_n$ (cosθ) are Legendre polynomials defined by

$$P_n(x) = \frac{1}{n2^n} \frac{d^n}{dx^n} [(x^2-1)^n] .$$

The most significant departure from the spherically symmetric field comes from the n=2 term, which corrects for most of the effects of the equatorial bulge. Therefore

$$PE = -GMm/r\,[1 - J_2\,(R/r)^2\,(3\cos^2\theta - 1)/2 + ...]$$

where $J_2$ = 1082.64x10**-6. At the poles $P_2$ = 2 and at the equator $P_2$ = -1. The coefficients for the higher zonal harmonics are three orders of magnitude reduced from the coefficient of the second zonal harmonic.

Equatorial bulge primarily makes the angle of the ascending node vary with time.

## *11.5 Sun synchronous Polar Orbit*

The equatorial bulge primarily makes the angle of the ascending node vary with time. The variation is given by

$$d\Omega/dt = -3/2\,J_2\,(GM)^{1/2}\,R^2\,a^{-7/2}\,(1-\varepsilon^2)^{-2}\cos i$$

Through suitable selection of the orbital inclination i, the rotation of the orbital plane can be made to match the rotation of the earth around the sun, yielding an orbit that is sun synchronous. The negative sign indicates a retrograde orbit, one with the satellite moving opposite to the direction of the earths rotation. The rotation rate for sun synchronous orbit is given by

$$\Omega = 2\pi/365.24 \text{ radians/year} = 2 \times 10^{-7} \text{ rad/sec} .$$

which is approximately one degree per day. Such a rate is obtained by placing the satellite into an orbit with a suitable inclination; for a satellite at a height of 800 km (assuming the orbit is roughly circular so that a = r), we find i = 98.5 degrees which is a retrograde orbit inclined at 81.5 degrees. The inclination for sun synchronous orbits is only a weak function of satellite height; the high inclination allows the satellite to view almost the entire surface of the earth from pole to pole.

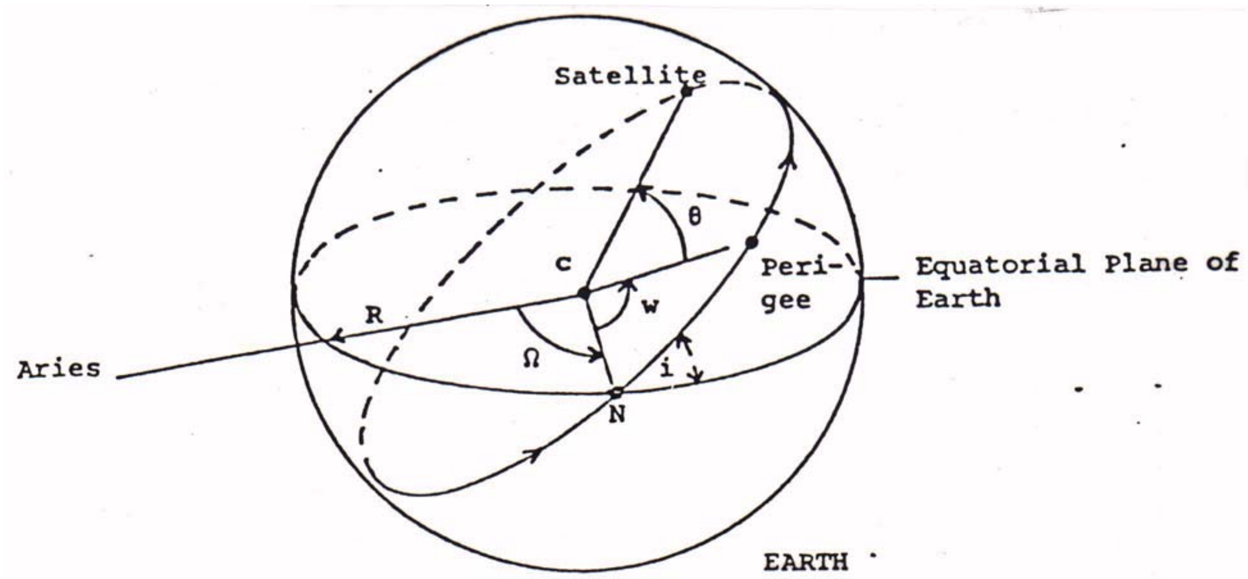


**Figure 11.1:** Orbital elements for an elliptical orbit showing the projection of the orbit on the surface of a spherical earth. C is the centre of the earth and R is the equatorial radius. i is the inclination of the orbit relative to the equatorial plane, Ω is the right ascension of the ascending node with respect to Aries, a is the semi-major axis of the ellipse, ε is the eccentricity, w is the argument of the perigee, and Θ is the angular position of the satellite in its orbit.

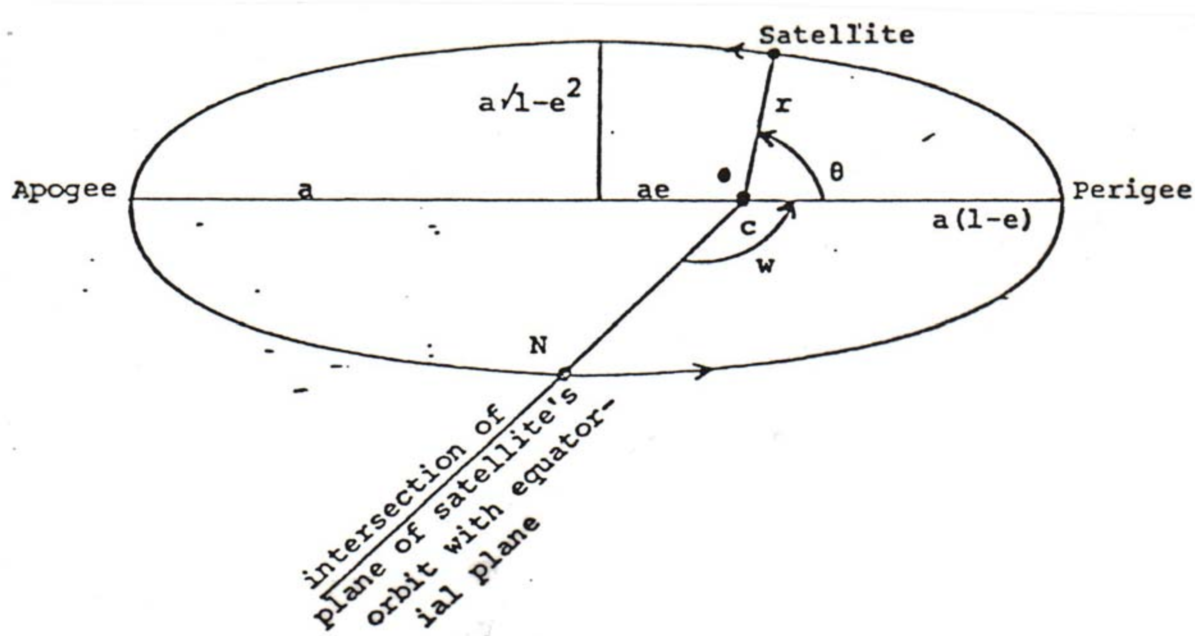


**Figure 11.2:** Elements of an elliptical orbit in the plane of the satellite orbit..