

CHAPTER 2

NATURE OF RADIATION

2.1 Remote Sensing of Radiation

Energy transfer from one place to another is accomplished by any one of three processes. Conduction is the transfer of kinetic energy of atoms or molecules (heat) by contact among molecules travelling at varying speeds. Convection is the physical displacement of matter in gases or liquids. Radiation is the process whereby energy is transferred across space without the necessity of transfer medium (in contrast with conduction and convection).

The observation of a target by a device separated by some distance is the act of remote sensing (for example ears sensing acoustic waves are remote sensors). Remote sensing with satellites for meteorological research has been largely confined to passive detection of radiation emanating from the earth/atmosphere system.

All satellite remote sensing systems involve the measurement of electromagnetic radiation. Electromagnetic radiation has the properties of both waves and discrete particles, although the two are never manifest simultaneously.

2.2 Basic Units

All forms of electromagnetic radiation travel in a vacuum at the same velocity, which is approximately 3×10^{10} cm/sec and is denoted by the letter c . Electromagnetic radiation is usually quantified according to its wave-like properties, which include intensity and wavelength. For many applications it is sufficient to consider electromagnetic waves as being a continuous train of sinusoidal shapes.

If radiation has only one colour, it is said to be monochromatic. The colour of any particular kind of radiation is designated by its frequency, which is the number of waves passing a given point in one second and is represented by the letter f (with units of cycles/sec or Hertz). The length of a single wave (i.e., the distance between two successive maxima) is called the wavelength and is denoted by λ (with units of microns to meters).

For monochromatic radiation, the number of waves passing a fixed point in one second multiplied by the length of each wave is the distance the wave train travelled on one second. But that distance is equal in magnitude to the velocity of light c . Hence,

$$c = f \lambda = 3 \times 10^{10} \text{ cm/sec} .$$

Since the frequency of a wave, f , is equal to the number of wavelengths in a distance of 3×10^{10} cm, it is usually a large number. Therefore, it is often much more convenient (especially in the infrared region of the spectrum) to consider the number of waves in one centimeter, called the wavenumber. This new unit the wavenumber is designated by ν (cm^{-1}) and

$$\nu = 1 / \lambda (\text{cm}^{-1}).$$

However, wavenumber becomes a relatively small unit in the microwave region; so, we use frequency in giga-Hertz units in that region. Similarly, because centimetres are usually too large for wavelength units, we use micrometers (μm) for most of the spectrum and for the very short wavelength region we use Angstrom units (\AA). Conversion of wavelength units is demonstrated below:

$$1 \text{ \AA} = 10^{-10} \text{ m} = 10^{-8} \text{ cm} = 10^{-4} \text{ \mu m}, \text{ and}$$

$$1 \text{ \mu m} = 10^{-6} \text{ m} = 10^{-4} \text{ cm} = 10^4 \text{ \AA}.$$

Conversion of frequency units follows:

$$1 \text{ cm}^{-1} = 3 \times 10^{10} \text{ Hz} = 30 \text{ GHz}, \text{ and}$$

$$1 \text{ GHz} = 10^9 \text{ Hz} = 1/30 \text{ cm}^{-1}.$$

Table 2.1 summarizes the regions and units of the electromagnetic spectrum commonly encountered in satellite meteorology.

2.3 *Definitions of Radiation*

The rate of energy transfer by electromagnetic radiation is called the radiant flux, which has units of energy per unit time. It is denoted by

$$F = dQ / dt$$

and is measured in joules per second or watts. For example, the radiant flux from the sun is about $3.90 \times 10^{26} \text{ W}$.

The radiant flux per unit area is called the irradiance (or radiant flux density in some texts). It is denoted by

$$E = dQ / dt / dA$$

and is measured in watts per square metre. The irradiance of electromagnetic radiation passing through the outermost limits of the visible disk of the sun (which has an approximate radius of $7 \times 10^8 \text{ m}$) is given by

$$E (\text{sun sfc}) = \frac{3.90 \times 10^{26}}{4\pi (7 \times 10^8)^2} = 6.34 \times 10^7 \text{ W m}^{-2}.$$

The solar irradiance arriving at the earth can be calculated by realizing that the flux is a constant, therefore

$$E (\text{earth sfc}) \times 4\pi R_{\text{es}}^2 = E (\text{sun sfc}) \times 4\pi R_{\text{s}}^2,$$

where R_{es} is the mean earth to sun distance (roughly $1.5 \times 10^{11} \text{ m}$) and R_{s} is the solar radius. This yields

$$E (\text{earth sfc}) = 6.34 \times 10^7 (7 \times 10^8 / 1.5 \times 10^{11})^2 = 1380 \text{ W m}^{-2}.$$

The irradiance per unit wavelength interval at wavelength λ is called the monochromatic irradiance,

$$E_{\lambda} = dQ / dt / dA / d\lambda,$$

and has the units of watts per square metre per micrometer. With this definition, the irradiance is

readily seen to be

$$E = \int_0^{\infty} E_{\lambda} d\lambda .$$

In general, the irradiance upon an element of surface area may consist of contributions which come from an infinity of different directions. It is sometimes necessary to identify the part of the irradiance that is coming from directions within some specified infinitesimal arc of solid angle $d\Omega$. The irradiance per unit solid angle is called the radiance,

$$I = dQ / dt / dA / d\lambda / d\Omega,$$

and is expressed in watts per square metre per micrometer per steradian. This quantity is often also referred to as intensity and denoted by the letter B (when referring to the Planck function). Table 2.2 summarizes these definitions.

In order to express the relationship between irradiance and radiance quantitatively, it is necessary to define the zenith angle, θ , as the angle between the direction of the radiation and the normal to the surface in question. The component of the radiance normal to the surface is then given by $I \cos \theta$. The irradiance represents the combined effects of the normal component of the radiation coming from the whole hemisphere; that is,

$$E = \int_{\Omega} I \cos \theta d\Omega$$

where in spherical coordinates

$$d\Omega = \sin \theta d\theta d\phi .$$

Figure 2.1 illustrates the radiance geometry.

Radiation whose radiance is independent of direction is called isotropic radiation. In this case, the integration over $d\Omega$ can be readily shown to be equal to π so that

$$E = \pi I .$$

Radiance is often expressed per unit frequency, $I(f)$, rather than per unit wavelength, $I(\lambda)$. Since the transformation from wavelength to frequency is given by $f = c / \lambda$, it follows that

$$I(\lambda) = I(f) c / \lambda^2 .$$

$I(\lambda)$ must not be confused with $I(f)$: they are different quantities. More will be said about this later in this chapter.

2.4 Historical Development of Planck's Radiation Law

It had long been observed that the surface of all bodies at a temperature greater than absolute zero (0 K) emits energy, in the form of thermal radiation. These electromagnetic waves were thought to be due to the motion of electric charges near the surface of the radiating body.

The study of radiation focused on the properties of a hypothetical black body, which is characterized by (a) complete absorption of all incident radiation (hence the term black) and (b) maximum possible emission in all wavelengths in all directions. In other words, it is the perfect

absorber and emitter of all radiation.

Many attempts, both empirical and theoretical in approach, were made in the years up to and about 1900 to understand the black body spectrum. In 1879, Stefan had empirically found that the irradiance of a black body was related to temperature by the law

$$E = \sigma T^4,$$

where

$$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ deg}^{-4}.$$

In 1884, Boltzmann produced a theoretical derivation of this equation.

Earliest accurate measurements of monochromatic irradiance are credited to Lummer and Pringsheim in 1899. They observed the now well-known emission spectra for black bodies at several different temperatures (shown in Figure 2.2).

Thermodynamical reasoning, while not giving a complete answer, did predict two characteristic features of the radiation. Wien, in 1893, was able to show that the monochromatic radiance was related to temperature and wavelength by

$$I(\lambda) = f(\lambda T) / \lambda^5,$$

where the form of the single function $f(\lambda T)$ was not evaluated. Also, the peak emission wavelength of a black body was shown to be inversely proportional to temperature, so that

$$\lambda_{\text{max}} = \text{const} / T .$$

In this derivation, he considered a cylindrical cavity with reflecting walls, one of which is a movable piston, filled with radiation at a temperature T . Radiation was known to exert a pressure proportional to its energy density. Taking this system through a Carnot cycle, a relation between the work done by the radiation (expressed in terms of monochromatic radiance) and temperature was obtained.

To evaluate the function $f(\lambda T)$ some of the detailed properties of a black body must be taken into account. In 1900, Rayleigh and Jeans attempted to evaluate $f(\lambda T)$ by considering a cubical cavity containing electromagnetic standing waves with nodes at the metallic surfaces and the energy of these waves obeying the Boltzmann probability distribution. Assuming a continuum of energy states, the average total energy of this system can be expressed as

$$\epsilon_{\text{av}} = \left[\int_0^{\infty} \epsilon e^{-\epsilon / kT} d\epsilon \right] / \left[\int_0^{\infty} e^{-\epsilon / kT} d\epsilon \right]$$

where the Boltzmann constant is given by

$$k = 1.381 \times 10^{-23} \text{ J/deg.}$$

This leads to

$$f(\lambda T) = 2ck\lambda T ,$$

which is in agreement with experiment only for long wavelengths. At short wavelengths the monochromatic radiance becomes infinite (often referred to as the ultraviolet catastrophe). The form of $f(\lambda T)$ obtained by Rayleigh and Jeans was a necessary consequence of the theories of

classical physics, yet it failed!

The discrepancy between experiment and theory was resolved by Planck in 1901, but at the expense of some of the concepts known to classical physics. Assuming that electromagnetic harmonic oscillations can only exist in quanta of hf (h is a constant, f is a frequency) and that oscillators emit energy only when changing from one to another of their quantized energy states, then the average total energy of this system is expressed as

$$\epsilon_{av} = \left[\sum_{n=0}^{\infty} nhf e^{-nhf/kT} \right] / \left[\sum_{n=0}^{\infty} e^{-nhf/kT} \right]$$

or

$$\epsilon_{av} = hf / (e^{hf/kT} - 1) .$$

This yields the proper form of $f(\lambda T)$,

$$f(\lambda T) = 2hc^2 / (e^{hc/\lambda kT} - 1) ,$$

and predicts the observed results if the Planck constant

$$h = 6.63 \times 10^{-34} \text{ J s} .$$

And the Planck law for radiation intensity (or monochromatic radiance) can be expressed as

$$B(\lambda, T) = c_1 / [\lambda^5 (e^{c_2/\lambda T} - 1)]$$

where

$$c_1 = 2hc^2 = 1.191044 \times 10^{-8} \text{ W/(m}^2 \text{ ster cm}^{-4})$$

and

$$c_2 = hc/k = 1.438769 \text{ K cm}$$

and $B(\lambda, T)$ has units of $\text{W/(m}^2 \cdot \text{ster} \cdot \text{cm)}$.

Written in terms of wavenumber rather than wavelength,

$$B(\nu, T) = c_1 \nu^3 / (e^{c_2 \nu / T} - 1)$$

where $B(\nu, T)$ has units of $\text{W/(m}^2 \text{ ster cm}^{-1})$. The difference in the Planck radiance curves when measured with respect to unit wavelength versus unit wavenumber is seen in Figure 2.3.

Thus, in the course of his successful attempt at resolving certain discrepancies between the observed energy spectrum of thermal radiation and the predictions of the classical theory, Planck was led to the idea that a system executing simple harmonic oscillations only can have energies which are integral multiples of a certain finite amount of energy (1901). A closely related idea was later applied by Einstein in explaining the photo-electric effect (1905), and by Bohr in a theory which predicted with great accuracy many of the complex features of atomic spectra (1913).

The work of these three physicists, plus subsequent developments by de Broglie, Schroedinger, and Heisenberg (ca. 1925), constitutes what is known as the quantum theory. Quantum theory and the theory of relativity together comprise the two most significant features of modern physics.

2.5 Related Derivations

2.5.1 Wien's Displacement Law

The peak of Planck function curve shifts to shorter wavelengths with an increase in temperature. The wavelength λ_{\max} for which the Planck function peaks at a given temperature T can be found from Planck's law by differentiating it with respect to λ and equating the result to zero. This yields the nonlinear equation

$$x = 5 (1 - e^{-x})$$

where $x = c_2 / (\lambda_{\max} T)$ whose solution is $x = 4.965114$. Therefore,

$$\lambda_{\max} = .2897 / T \text{ (cm)}$$

which is Wien's displacement law. This law indicates that the wavelength of maximum Planck radiance varies inversely with temperature. Note that when the Planck function is expressed in terms of wavenumber, differentiating with respect to ν and equating the result to zero yields $y = 3 (1 - e^{-y})$ where $y = c_2 \nu_{\max} / T$ with the solution $\nu_{\max} = 1.95 T \text{ (cm}^{-1}\text{)}$. It is important to realize that $\nu_{\max} \neq 1 / \lambda_{\max}$.

The Planck radiance at the Wien wavelength varies as temperature to the fifth power. To see this more clearly consider

$$\begin{aligned} B(\lambda_{\max}, T) &= c_1 / [\lambda_{\max}^5 (e^{c_2 / \lambda_{\max} T} - 1)] \\ &= c_1 T^5 / [(.2897)^5 (e^{c_2 / .2897} - 1)] \\ &= c_3 T^5 \end{aligned}$$

where c_3 is a constant. In a similar way, it can be shown that $B(\nu_{\max}, T) = c_4 T^3$.

The radiative temperature of the surface of the sun is roughly 5780 K. Applying Wien's Law at the sun's surface temperature, one finds the maximum Planck radiance to be at 0.50 μm , which is near the centre of the visible region of the spectrum. Since the sun radiates nearly as a black body, we can say that the solar energy reaching the earth is a maximum in the visible region.

On the other hand, the earth's atmospheric temperature averaged vertically is around 255 K; therefore, the maximum emitted energy of the earth's atmosphere occurs roughly at 11 μm , that is, well into the infrared region. In fact, if the black body curves (Planck functions) for temperatures of 255 K and 5780 K are plotted next to each other, the curves are almost entirely separate. Thus, the spectral distribution of the incoming solar radiation is quite different from that of the outgoing terrestrial radiation (see Figure 2.4).

Wien's displacement law derives its name from the fact that as the temperature increases, the point of maximum intensity of the black body curve is displaced toward the shorter wavelengths. Since the wavelength of the maximum value determines the colour perceived while observing the complete spectrum, we have an explanation for the transition in colour of a heated iron bar from red to a white glow with increased heat. As the temperature is raised the longer wavelength red light becomes visible first. Then higher temperatures make additional wavelengths visible. Finally, when the temperature is sufficiently high, the radiation consists of a mixture of all the visible wavelengths and, hence, appears white hot. For similar reasons, the filament of an incandescent lamp must be heated to thousands of degrees K in order to be an efficient emitter of visible light, while infrared lamps operate at lower temperatures.

2.5.2 Rayleigh-Jeans Radiation Law

In the microwave region of the electromagnetic spectrum, when $\lambda > 0.5$ cm and T is at terrestrial temperatures, the exponent in the Planck function is small, and hence one can make the approximation

$$e^{c_2/\lambda T} = 1 + c_2/\lambda T$$

that yields the following asymptotic expansion:

$$B(\lambda, T) = c_1 T / (c_2 \lambda^4)$$

Similarly,

$$B(\nu, T) = c_1 \nu^2 T / c_2$$

for sufficiently small ν . These formulas represent the Rayleigh-Jeans law of radiation and the spectral region $\lambda > 0.5$ cm is called the Rayleigh-Jeans region in atmospheric physics. Note that in the Rayleigh-Jeans region the Planck function is linear to T. For $\lambda T > 10$ cm K, the Rayleigh-Jeans approximation is within 2% or better.

2.5.3 Wien's Radiation Law

In the near infrared region and beyond into the visible and ultraviolet regions, i.e., $\lambda < 10^{-3}$ cm, and when T is at terrestrial temperatures, the exponent in the Planck function is large and, hence

$$e^{c_2/\lambda T} \gg 1.0$$

Consequently, the constant 1.0 can be ignored in the denominator to yield another asymptotic expansion for the Planck function, namely,

$$B(\lambda, T) = c_1 \lambda^{-5} e^{-c_2/\lambda T},$$

or

$$B(\nu, T) = c_1 \nu^3 e^{-c_2 \nu/T}$$

for sufficiently large ν . These are two forms of Wien's law of radiation. The spectral region $\lambda < 10^{-3}$ cm is called the Wien region when dealing with atmospheric temperatures. In the Wien region, the Planck function is highly nonlinear in temperature. For $\lambda T < 0.5$ cm K, the Wien approximation is within 2% or better.

2.5.4 Stefan-Boltzmann Law

The black body irradiance is obtained by integrating the Planck function over all wavelengths and angles,

$$E = \int_0^{\infty} E_{\lambda} d\lambda = \pi \int_0^{\infty} \frac{c_1 d\lambda}{\lambda^5 [e^{c_2/\lambda T} - 1]},$$

Let $x = c_2/\lambda T$ then

$$E = \frac{\pi c_1 T^4}{c_2^4} \int_0^{\infty} \frac{x^3 dx}{(e^x - 1)} = \frac{\pi^5 c_1 T^4}{15 c_2^4} = \sigma T^4 .$$

Note that

$$\int_0^{\infty} B(\lambda, T) d\lambda = \int_0^{\infty} B(\nu, T) d\nu ,$$

which implies for a given temperature

$$B(\lambda, T) = \nu^2 B(\nu, T) .$$

It is often useful to know the fraction of the total blackbody radiance coming from below a specific wavelength, expressed by

$$\int_0^{\lambda} E_{\lambda} d\lambda / \sigma T = \pi \int_0^{\lambda} B_{\lambda} d\lambda / \sigma T .$$

Figure 2.5 shows the fraction of total blackbody radiance emitted below a specific wavelength as a function of λT . Note that for $\lambda = \lambda_{\max}$ the fraction is .23, so less than 25% of the radiance comes from wavelengths shorter than the peak wavelength. Figure 2.5 illustrates that less than 1% of the solar energy comes from wavelengths longer than 4 microns, and less than 1% of terrestrial energy from a blackbody at 250 K comes from wavelengths shorter than 4 microns.

2.5.5 Brightness Temperature

Ultimately, one is interested in the temperature that corresponds to a particular Planck function value B_{λ} . This temperature is determined by inverting the Planck function,

$$T = c_2 / \left[\lambda \ln \left(\frac{c_1}{\lambda^5 B_{\lambda}} + 1 \right) \right] = c_2 \nu / \left[\ln \left(\frac{c_1 \nu^3}{B_{\nu}} + 1 \right) \right]$$

The temperature derived is called brightness temperature because of its historical connection with radio astronomy; however, the terms radiance temperature or equivalent black body temperature are also frequently used.

In the Rayleigh-Jeans regions, one can write

$$T = (c_2/c_1) \lambda^4 B_{\lambda} = (c_2/c_1) B_{\nu} / \nu^2 ,$$

where $c_2/c_1 = 1.208021 \times 10^5$. On the other hand in the Wien region

$$T = c_2 / \left[\lambda \ln \left(\frac{c_1}{\lambda^5 B_{\lambda}} \right) \right] = c_2 \nu / \left[\ln \left(\frac{c_1 \nu^3}{B_{\nu}} \right) \right]$$

where $B(\nu, T)$ has units of $W/(m^2 \text{ ster } cm^{-1})$.

As we have seen from Planck's Law, as temperature increases the radiance also increases; the percentage increase varies as a function of wavelength and temperature. The

percentage change in radiance to a corresponding percentage change in temperature, called temperature sensitivity, α , of a given spectral band is defined as $dB/B = \alpha dT/T$. For infrared wavelengths, we find that $\alpha \approx c_2\nu/T = c_2/\lambda T$. Thus the larger wavenumbers (shorter wavelengths) have a greater temperature sensitivity than the smaller wavenumbers (longer wavelengths). Consider the two infrared windows for example; at 300 K, the temperature sensitivity for 900 cm^{-1} (longwave window at 11 μm) is 4.3 and for 2500 cm^{-1} (shortwave window at 4 μm) it is 12. The temperature sensitivity indicates the power to which the Planck radiance depends on temperature, since B proportional to T^α satisfies the equation. Thus the radiance in the shortwave window is varying roughly as temperature to the twelfth power and in the longwave window roughly as temperature to the fourth power. Table 2.3 indicates the temperature sensitivity of some of the other spectral regions.

Table 2.1 Regions and Units of the Electromagnetic Spectrum

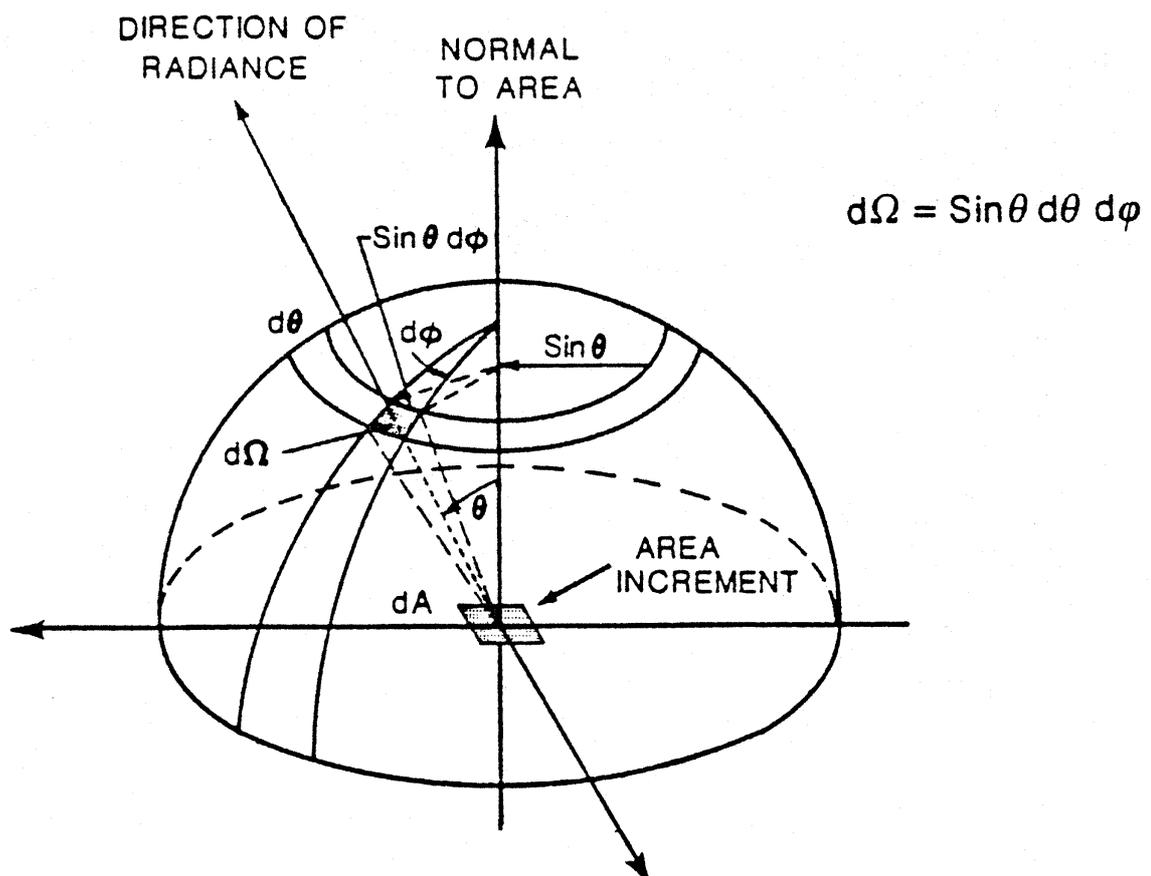
WAVELENGTH			FREQUENCY		WAVENUMBER
cm	μm	\AA	Hz	GHz	cm^{-1}
10^{-5}	0.1	1,000	3×10^{15}		
Near Ultraviolet (UV)					
4×10^{-5}	0.4	4,000	7.5×10^{14}		
Visible					
7.5×10^{-5}	0.75	7,500	4×10^{14}		13,333
Near Infrared (IR)					
2×10^{-3}	20	2×10^5	1.5×10^{13}		500
Far Infrared (IR)					
0.1	10^3		3×10^{11}	300	10
Microwave (MW)					

Table 2.2 Definitions of Radiation

QUANTITY	SYMBOL	UNITS
Energy	DQ	Joules
Flux	dQ/dT	Joules/sec = Watts
Irradiance	$dQ/dT/dA$	Watts/metre**2
Monochromatic Irradiances	$dQ/dT/dA/d\lambda$	$\text{W/m}^{**2}/\text{micron}$
	or $dQ/dT/dA/d\nu$	$\text{W/m}^{**2}\text{cm}^{-1}$
Radiance	$dQ/dT/dA/d\lambda/d\Omega$	$\text{W/m}^{**2}/\text{micron}/\text{ster}$
	or $dQ/dT/dA/d\nu/d\Omega$	$\text{W/m}^{**2}/\text{cm}^{-1}/\text{ster}$

Table 2.3 Temperature sensitivity of various spectral regions.

Wavenumber	Typical Scene Temperature	Temperature Sensitivity
700	220	4.58
900	300	4.32
1200	300	5.76
1600	240	9.59
2300	220	15.04
2500	300	11.99

**Figure 2.1:** The Radiance Geometry

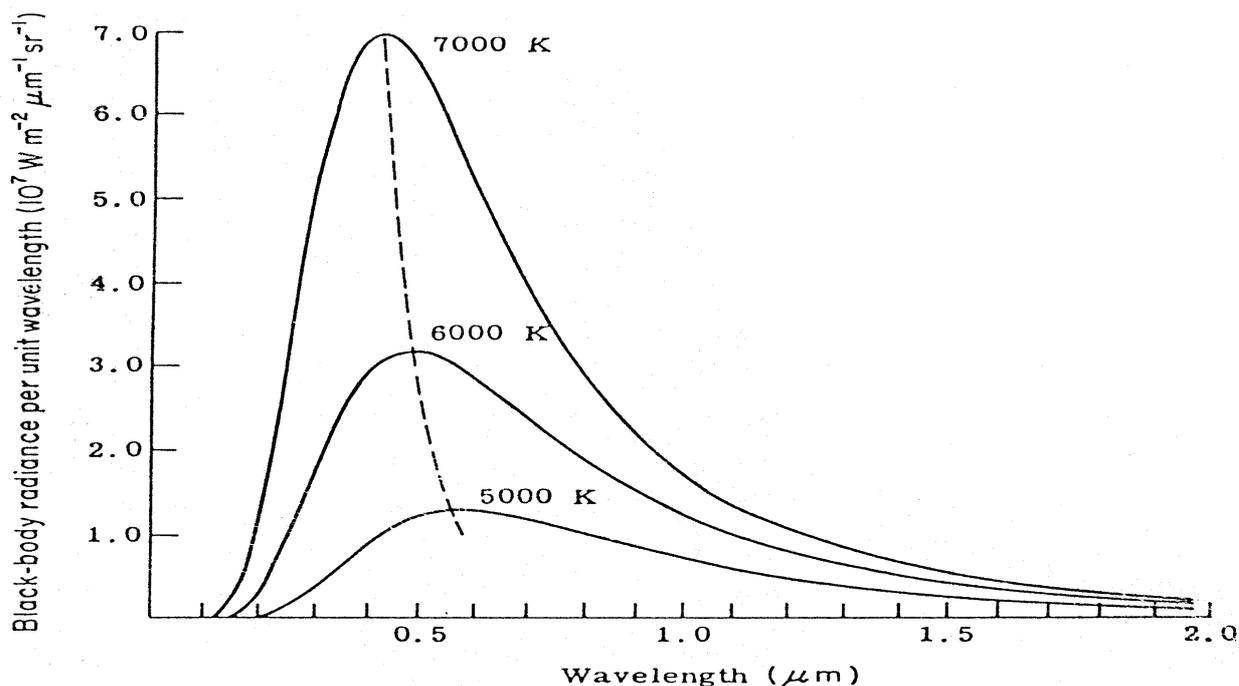


Figure 2.2: Emission Spectra for Black Bodies with the Indicated Temperatures

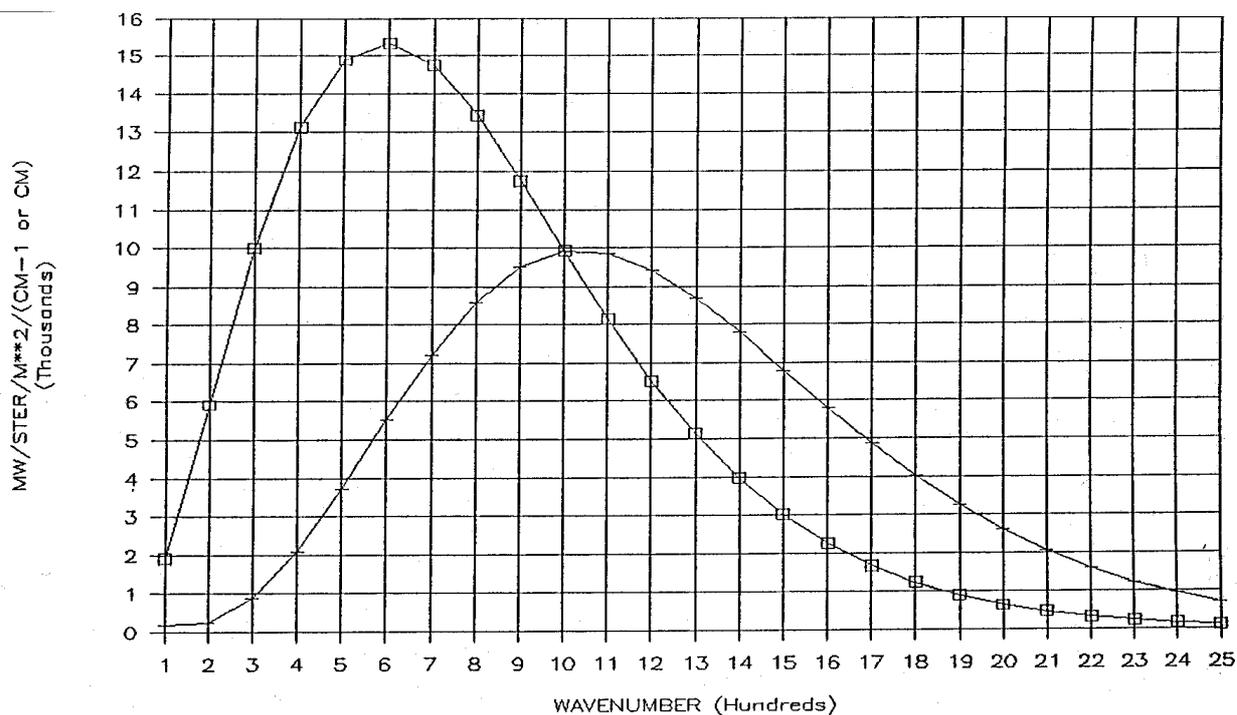


Figure 2.3: Planck radiance curves at 300 K. + indicates $B(\lambda, T)$ in units of $\text{mW/ster/m}^2/\text{cm}$ (per unit wavelength) and \square indicates $B(\nu, T)$ in units of $\text{mW/ster/m}^2/\text{cm}^{-1}$ (per unit wavenumber). $B(\lambda, T)$ is shown multiplied by a factor of 10^{-7} (e.g. $B(10 \mu\text{m}, 300 \text{ K})$ is 9.9×10^7) while $B(\nu, T)$ is shown multiplied by a factor of 10^{-1} (e.g. $B(1000 \text{ cm}^{-1}, 300 \text{ K})$ is 99.0). Note that Wiens law for $B(\nu, T)$ becomes $\nu(\text{max}) = 1.95T \text{ (cm}^{-1}\text{)}$ while the more familiar form for $B(\lambda, T)$ is $\lambda(\text{max}) = .2897/T \text{ (cm)}$.

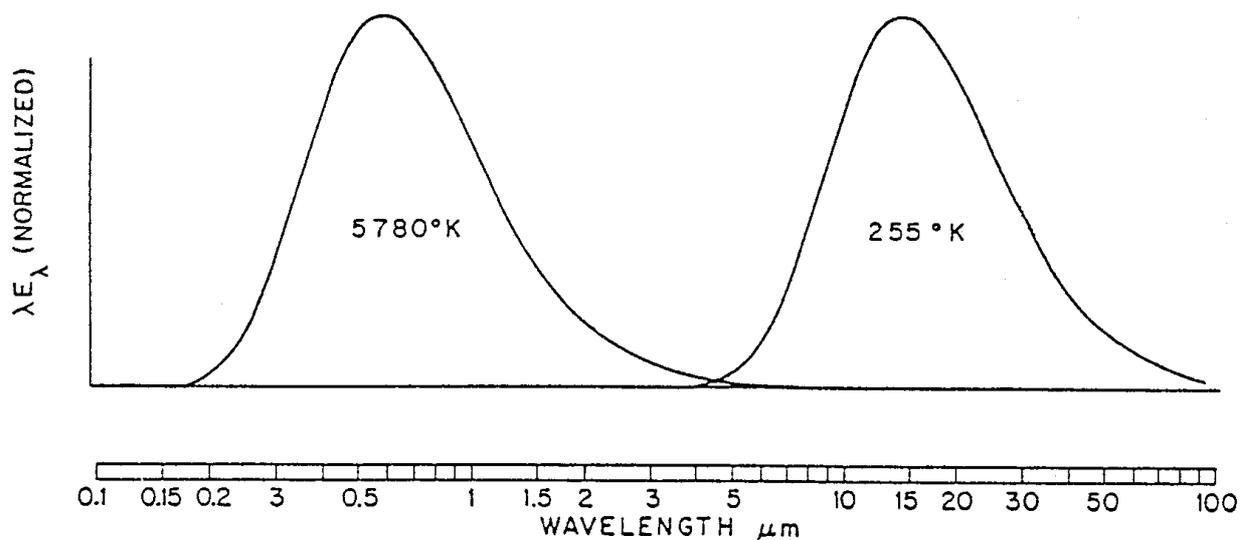


Figure 2.4: Normalized black body spectra representative of the sun (left) and earth (right), plotted on a logarithmic wavelength scale. The ordinate is multiplied by wavelength so that the area under the curves is proportional to irradiance.

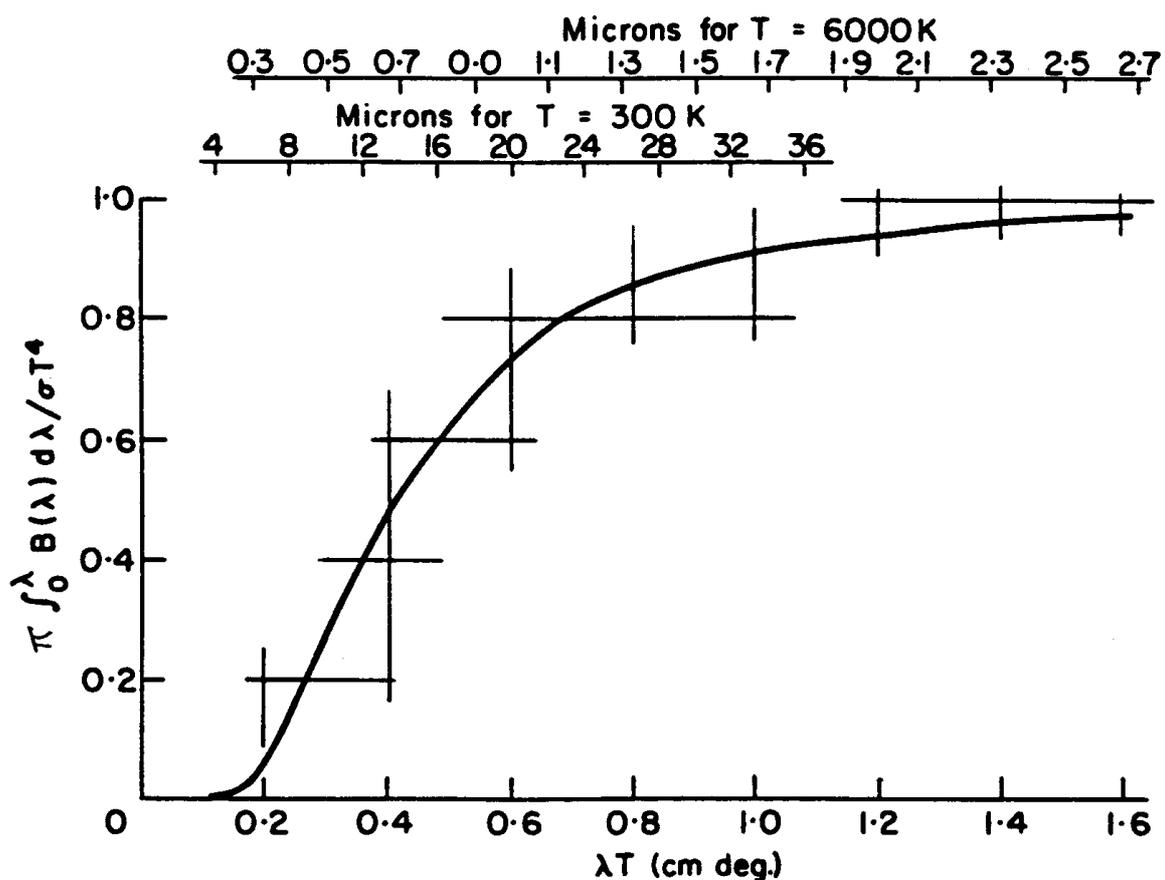


Figure 2.5: Fraction of total blackbody radiance coming from below a given wavelength for a given temperature.