Can we reconstruct localized features from non-local observations?
The role of observation and background errors

Olaf Stiller
Deutscher Wetterdienst, Data Assimilation Unit, Frankfurter Str. 135, Offenbach, Germany

Observation subset
exact reconstruction
to both levels
weighting factors

(smear)
(filter)
G
observation subsets

⊂ {smear, filter}
for which

DA systems have to deal with strongly localised features like cloud tops, inversions, etc.

The main mathematical result

A novel way of writing the costfunction minimum \( x^o \) has been presented - see Eq (4).

This expands \( x^o \) as a sum over Pseudo Inverses (PIs) (see Eq (4)), each corresponding to a different subset \( \tau \) of the available observations.

(\( \tau = (\tau_1, \tau_2, ... , \tau_p) \subset \{1,2,...,p\} \))

The role of the Pseudo Inverse (PI)

The PI for a given subset \( \tau \) leads to an analysis state which is completely consistent with all the observations from \( \tau \).

It can therefore be regarded as a direct transformation of the observations \( \tau \) into model space.

However, the PI is generally not optimal:

- The PI neglects observation error
- The PI tends to amplify noise

The coefficients \( G_\tau \) in the expansion (4) show to which extent different observation sets \( \tau \) contribute to the analysis increments \( x^o \). There are two limiting cases: They are smaller the more the observation operators \( \tau \) overlap.

(3) Finite observation errors degrade representation of localised features

The general solution (2) for the cost function minimum can be written as

\[
x^o = \sum \frac{G_\tau x^o_\tau}{G_\tau}
\]

with \( x^o_\tau \) : PI corresponding to observation subset

\( \tau = (\tau_1, \tau_2, ... , \tau_p) \subset \{1,2,...,p\} \)

\[
\sum \frac{G_\tau}{G_\tau}
\]

are smaller the more the observation errors \( r_j \)

are smaller the more the observation operators \( H \), overlap

\( (G_\tau x^o_\tau) \) reduces to zero if \( (H BH^T)\) does not exist.

(4) Example: A simple model problem

The PI related to both observations \( (\tau = (\{1\}, \{2\}) \)

yields an exact reconstruction (assuming obs errors are sufficiently small)

\[
x^o_{\{1,2\}} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}
\]

Single observation PIs, on the other hand, smear out the signal from one observation to both levels by distributing it statistically according to \( H \) and \( B \).

\[
x^o_{\{1\}} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}
\]

The weighting factors \( G_{\{1\}} \) and \( G_{\{2\}} \) act as a filter. The 2 obs PI \( x^o_{\{1,2\}} \) amplifies noise particularly when \( \det(B) \) or \( h_2 \) are very small. One has, e.g.,

\[
G_{\{1,2\}} x^o_{\{1,2\}} \rightarrow h_2
\]

as \( h_2 \rightarrow 0 \)

(5) Summary and conclusions

The expansion of \( x^o \) in terms of PIs (see Eq. (4))

- The coefficients \( G_\tau \) filter the noise.
- \( G_\tau \) is very small if observation errors exceed the required precision.

- The coefficients \( G_\tau \) are

\[
G_\tau = \frac{\det(B_\tau)}{\prod_{\tau_i \neq \tau} R_{\tau_i}}
\]

- \( B_\tau \) : background correlation matrix in obs space.

- \( \det(B_\tau) \) gives a measure for the overlap of obs-operators.

- Normalized obs error

Conclusions

- The expansion of \( x^o \) in terms of PIs shows to which extent measured degrees of freedom (which are non-local) are exploited for reconstructing spatial features.

- Large obs errors imply degradation spatial resolution (not only decrease weight of obs in assimilation process)

Reconstruction of localized features

- requires small obs errors.

- Obs errors have to be smaller the more observation operators overlap.

- observations contradict statistical expectations from B matrix.

For proofs and further discussion:

O. Stiller, The role of observation and background errors for reconstructing localized features from non-local observations.,


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